MODEL OF TURBULENT MOTION IN AN INCOMPRESSIBLE LIQUID IN APPARATUS WITH A PERMEABLE BARRIER

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A model of turbulent motion of an incompressible fluid in apparatus with a stationary packed bed is presented. The model is constructed using modern ideas about the mechanics of interpenetration continua and semiempirical theories of turbulence. The computational results are in satisfactory agreement with experiment and explain the well-known appearance of macroscopic nonuniformities in the velocity profiles behind a porous medium.

Apparatus with a permeable barrier (stationary packed bed, packet of grids, porous insert) is widely employed in different technological processes [1]. A distinguishing feature of their aerodynamics is that the liquid (gas) completely flows from one chamber into another through a porous medium. In the process, motion in a permeable barrier significantly affects the structure of flow in the free parts of the apparatus. Stream and detached flows predominate in the barrier, because the pore area and directions of flow change repeatedly. According to the experimental data [2-5], a high degree of turbulence is reached in the pores of a packed bed as a result of eddy formation, caused by detachment of streams, which then break apart between the grains. The turbulent parcels consist of elements with a wide range of linear sizes, and in a macroscopic volume the generation and dissipation of turbulence energy are locally in equilibrium. Following Khintse [6], it can be assumed that intense inflow of energy into the cascade process is determined by the largest turbulence elements, whose sizes are associated with the scales of the average motion. In [7] a relation between the kinetic energies of the pulsational and average motions in a packed bed was proposed in the form

$$\langle v^2\rangle = \frac{1-\varepsilon}{2}|\mathbf{V}|^2 \tag{1}$$

where ε is the porosity of the packed bed. This relation was used to calculate the molar component of the effective thermal conductivity (diffusion coefficient); the calculations agreed satisfactorily with the experimental data.

Intense dissipation of the kinetic energy of turbulence occurs in the pore space. In addition, inertial forces in the stream and detached flows play the main role. For this reason, according to the hypothesis of [8], it can be assumed that the dissipation of energy of pulsational motion depends on the kinetic energy of turbulence k and the average linear size of the eddies, which is associated with the scale of the average flow. In porous media the characteristic size is the diameter of the elements of the packed bed. Therefore the dissipation of the kinetic energy of turbulence can be represented in the form

$$\bar{\varepsilon} = \left[\frac{1}{4}(1-\varepsilon)\right]^{1/2}\frac{k^{3/2}}{d_3}, \tag{2}$$

where $k = \langle v^2\rangle/2$ and d_3 is the diameter of an element of the packed bed.

From Eqs. (1) and (2) we can determine the coefficient of effective viscosity in a permeable barrier, using the standard formula of the $k - \varepsilon$ model, i.e.,

$$\nu_t = 0{,}09k^2/\bar{\varepsilon} = 0{,}09|\mathbf{V}|d_3. \tag{3}$$

In this form the expression for ν_t is identical to the algebraic model of turbulence in porous media and it agrees, to within the empirical constant $C_\mu = 0.09$, with the experimental data [7, 9, 10].

A somewhat different method was proposed in [11] for finding the relation between the characteristics of the pulsational and average motion in porous media. In [11] the Kolmogorov microscales of velocity and length, corresponding to the universal theory of equilibrium,

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when the viscosity of the liquid plays the main role in the dissipating extremely small scales [6], were used.

Thus using the algebraic model (1)-(3) to describe the turbulent motion in the porous medium G_2 and the well-known $k-\varepsilon$ model of turbulence in the free parts G_1 and G_3, we can invoke the main assumptions of the mechanics of interpenetrating continua for calculating aerodynamics of reactors with a permeable barrier. We present below some results of the calculation of the turbulent motion of an incompressible liquid.

We consider flow in apparatus with a flat or axisymmetric construction. We assume that the motion is stationary, the characteristics of the porous medium are given, and the resistance of the medium obeys a nonlinear filtration law. We include the effect of the gratings, bounding the packed bed in the resistance of the porous medium, in the resistance [12]. We orient the Ox_1 axis of a Cartesian coordinate system along the axis of the apparatus and the Ox_2 axis orthogonal to the Ox_1 axis. We write in the following form the system of generalized equations of turbulent motion and continuity, derived by the method of averaging over the liquid phase of a local volume of the porous medium, using the Boussinesq hypothesis about the effective viscosity:

$$u_1\frac{\partial u_1}{\partial x_1}+u_2\frac{\partial u_1}{\partial x_2}=-\frac{\partial p}{\partial x_1}+2\frac{\partial}{\partial x_1}\left(\nu_{\text{eff}}\frac{\partial u_1}{\partial x_1}\right)+\frac{1}{x_2^j}\frac{\partial}{\partial x_2}\left[x_2^j\nu_{\text{eff}}\left(\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}\right)\right]-\xi u_1; \tag{4}$$

$$u_1\frac{\partial u_2}{\partial x_1}+u_2\frac{\partial u_2}{\partial x_2}=-\frac{\partial p}{\partial x_2}+\frac{\partial}{\partial x_1}\left[\nu_{\text{eff}}\left(\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}\right)\right]+\frac{2}{x_2^j}\frac{\partial}{\partial x_2}\left(x_2^j\nu_{\text{eff}}\frac{\partial u_2}{\partial x_2}\right)-\frac{2\nu_{\text{eff}}u_2 j}{x_2^2}-\xi u_2; \tag{5}$$

$$\frac{\partial \varepsilon u_1 x_2^j}{\partial x_1}+\frac{\partial \varepsilon u_2 x_2^j}{\partial x_2}=0, \tag{6}$$

where $j = 0, 1$, respectively, for the flat and axisymmetric cases; x_1 and x_2 are the longitudinal and transverse coordiantes; u_1 and u_2 are the longitudinal and transverse components of the velocity; p is the pressure; $\xi=\xi_1+\xi_2|V|$; $\xi_1=150(1-\varepsilon)^2H_0^2/(\varepsilon^2 d_3^2\text{Re})$; $\xi_2=1.75(1-\varepsilon)H_0/(\varepsilon d_3)$; $\text{Re} = u_0H_0/\nu$ (H_0 is the half-width or radius of the entrance section of the channel and u_0 is the average flow velocity); $\nu_{\text{eff}} = 1/\text{Re} + \nu_t$.

In the free parts of the channel the coefficient of effective viscosity is determined from the well-known $k-\varepsilon$ model of turbulence [13]:

$$u_1\frac{\partial k}{\partial x_1}+u_2\frac{\partial k}{\partial x_2}=\frac{\partial}{\partial x_1}\left(\nu_{\text{eff}}\frac{\partial k}{\partial x_1}\right)+\frac{1}{x_2^j}\frac{\partial}{\partial x_2}\left(x_2^j\nu_{\text{eff}}\frac{\partial k}{\partial x_2}\right)+P_k-\varepsilon-\frac{2k}{\text{Re}\,x_n^2}; \tag{7}$$

$$u_1\frac{\partial \varepsilon}{\partial x_1}+u_2\frac{\partial \varepsilon}{\partial x_2}=\frac{\partial}{\partial x_1}\left[\left(\frac{1}{\text{Re}}+\frac{\nu_t}{\sigma}\right)\frac{\partial \varepsilon}{\partial x_1}\right]+\frac{1}{x_2^j}\frac{\partial}{\partial x_2}\left[x_2^j\left(\frac{1}{\text{Re}}+\frac{\nu_t}{\sigma}\right)\frac{\partial \varepsilon}{\partial x_2}\right]+C_1\frac{\varepsilon}{k}P_k-\frac{\varepsilon}{k}\left(C_2 f_{\text{eff}}\varepsilon+\frac{2g_{\text{eff}}k}{x_n^2\,\text{Re}}\right). \tag{8}$$

Here $P_k=\nu_t\left\{2\left[\left(\frac{\partial u_1}{\partial x_1}\right)^2+\left(\frac{\partial u_2}{\partial x_2}\right)^2+j\left(\frac{u_2}{x_2}\right)^2\right]+\left(\frac{\partial u_2}{\partial x_1}+\frac{\partial u_1}{\partial x_2}\right)^2\right\}$; $g_{\text{eff}}=\exp(-C_3 v_* x_n \text{Re})$; $f_{\text{eff}} = 1 - 0.22 \times \exp\left[-\left(\frac{k^2}{6\varepsilon}\text{Re}\right)^2\right]$; $\nu_t=C_\nu\frac{k^2}{\varepsilon}$; $C_1=1.35$; $C_2=1.8$; $C_3=0.5$; $\sigma=1.3$; $C_\nu=0.09[1-\exp(-0.0115 v_* x_n \text{Re})]$; x_n is the distance along the normal from the wall; and, v_* is the dynamic velocity.

The kinetic energy of turbulence and its dissipation and the coefficient of effective viscosity in the permeable barrier are found from the expressions (1)-(3), respectively.

Simultaneous analysis of Eqs. (1)-(8) makes it possible to close the system of equations of motion and to describe, on the basis of the same assumptions, the flow in all parts of the reactor. The conditions of matching at the interfaces between the media [14]

$$\left(p+\frac{u_1^2+u_2^2}{2}-2\nu_{\text{eff}}\frac{\partial u_1}{\partial x_1}\right)_-=\left(p+\frac{u_1^2+u_2^2}{2}-2\nu_{\text{eff}}\frac{\partial u_1}{\partial x_1}\right)_+,$$

$$\left[\nu_{\text{eff}}\left(\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}\right)\right]_-=\left[\nu_{\text{eff}}\left(\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}\right)\right]_+,\quad (u_1\varepsilon)_-=(u_1\varepsilon)_+,\quad (u_2)_-=(u_2)_+$$

were used to construct a ripple-through computational scheme.

The problem was solved with the following boundary conditions. The distribution of the velocity and turbulent characteristics, corresponding to developed flow in the channel, were prescribed at the entrance (Γ_1, Fig. 1a): $u_1 = u_{10}(x_2)$, $k = k_0(x_2)$, $\varepsilon = \varepsilon_0(x_2)$, $u_2 = 0$.

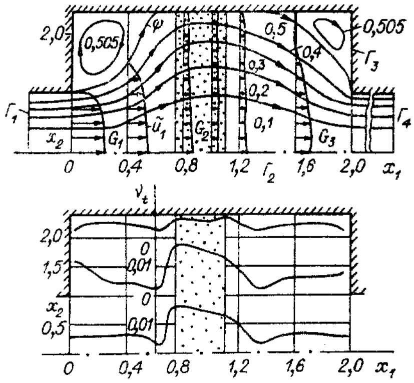

Fig. 1

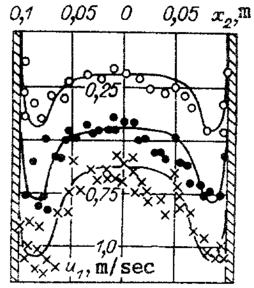

Fig. 2

Conditions of symmetry were used on the axis Γ_2: $\partial u_1/\partial x_2 = 0$, $\partial k/\partial x_2 = 0$, $\partial \varepsilon/\partial x_2 = 0$, $u_2 = 0$. The attachment condition was imposed on the walls Γ_3 outside the porous medium: $u_1 = u_2 = k = \varepsilon = 0$; conditions of slipping and impermeability for tangential and normal components of the velocity, respectively, were imposed on the surfaces of the apparatus bounding the packed bed. Soft conditions, corresponding to stabilized flow, were imposed on the exit section Γ_4.

The system of equations of motion and continuity was solved by a numerical method in the variables ω and ψ, the vorticity and the stream function, respectively [14]. The transport equations for the kinetic energy of turbulence and its dissipation were solved by the Gauss–Seidel method in the free parts of the channel, using the algebraic expressions for these quantities in the porous medium (1) and (2). The difference grid had dimensions of 41 × 41 and a nonuniform step, decreasing near the wall, was used. All calculations were performed on a BÉSM-6 computer.

The main process parameters of the problem are the Reynolds number, the resistance of the layer, and the geometric dimensions of the apparatus with a porous insert. The porosity of the medium was mainly assumed to be constant, corresponding to cubic or random packing. In some calculations the distribution of ε along the normal to the wall was found using the formula [15]

$$\varepsilon = \varepsilon_0[1 - s\exp(-x_n/d_3)] \tag{9}$$

where ε_0 is the porosity at the center and the coefficient s is related with ε_0, in order to take into account the nonuniformity of the porosity near the wall.

The proposed model was verified on experimental data obtained in a flow-through reactor with a stationary packed bed for synthesis of ammonia [16]. The profiles of the longitudinal velocity were found behind the porous medium for three values of the pressure drop $(0.1, 0.2, 0.3)\cdot 10^5$ N/m^2. It is known [10] that under natural conditions (in experiments) the porosity of the bed increases somewhat near a bounding wall, and for this reason in the calculations the distribution of ε was given by the formula (9). In Fig. 2 the computed profiles of the longitudinal velocity are compared with the experimental data. Their satisfactory agreement indicates that the model describes the macroscopic nonuniformity in the velocity profiles observed in numerous experiments.

The calculations of turbulent flow were performed for two types of apparatus with a permeable barrier. In all cases, in order to simplify the analysis, the porosity of the packed bed was assumed to be constant, corresponding to cubic packing. Figure 1a shows the pattern of streamlines and profiles of the longitudinal velocity in the reactor with concentrated inflow and outflow (Re = 4300, $\varepsilon = 0.476$, $d_3 = 0.0045$ m, $H_0 = 0.0225$ m). One can see that in front of the packed bed the liquid spreads out and completely fills the reactor chamber. The streamlines continue to expand inside the packed bed and, as the motion proceeds, start to change direction. Behind the bed they bunch up at the exit from the chamber, bending around the stagnation zone. It is easy to see that the streamlines are not orthogonal to the free boundaries of the packed bed, and this indicates that the liquid slips as it flows in and out of the porous medium. The profiles $\tilde{u}_1 = \varepsilon u_1$ become significantly deformed. In particular, they are redistributed inside the packed bed also. The distribution of the kinetic energy k in the direction of flow expresses most vividly the change in the turbulent characteristics of the flow in the reactor chamber (Fig. 3a). The values of the kinetic energy of turbulence were constructed in logarithmic coordinates, in view of the fact that they change significantly in the region of flow. In the free parts of the apparatus the values of k are

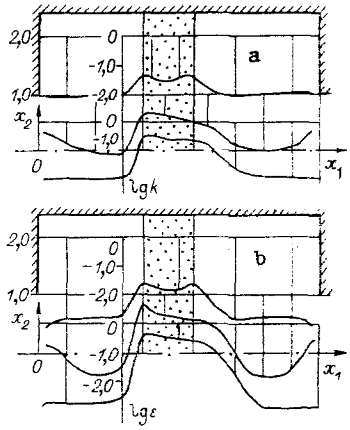

Fig. 3

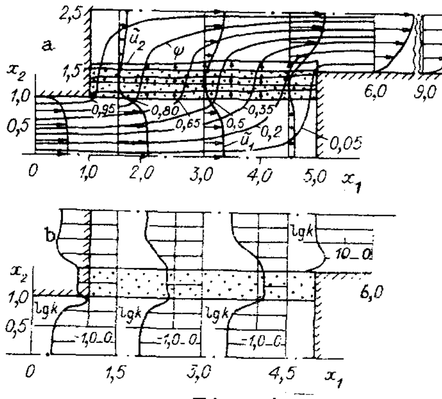

Fig. 4

not very large, and k changes quite sharply only near the entrance and exit boundaries of the chamber (in a narrow zone with large velocity gradients). In the packed bed k is determined by the formula (1) and reaches 30% of the kinetic energy of the average motion. The deformation of velocity profiles caused by the spreading of liquid in front of the packed bed generates turbulence energy and increases k. Intense eddy formation at the exit from the porous medium maintains a high value of k, and then the turbulence collapses. The character of the changes in the dissipation of turbulence energy is shown in Fig. 3b. One can see that intense dissipation of turbulence energy occurs in the porous medium.

The distribution of the effective viscosity ν_t is analogous to the change in the turbulent characteristics (see Fig. 1b). Note the increase in ν_t in the flow in front of the bed as a result of spreading of the liquid in front of the insert. The value of ν_t in the porous medium was found using the formula (3) and was proportional to the filtration velocity.

In the calculations we studied the effect of the Reynolds number, the ratio of the diameter of the chamber to the diameter of the inflow (outflow) unit, and the resistance of the bed on the flow patterns. As a result, it was shown that increasing Re and the ratio of the diameters incresaes the nonuniformity of the distribution of the filtration velocity over the section of the bed, while increasing the resistance of the bed results in a more uniform distribution of flow in the porous medium.

The results of the calculation of turbulent motion of an incompressible liquid in a flat apparatus with a Z-shaped flow pattern are presented in Fig. 4. The calculation was performed for the following values of the process parameters: Re = 40,000, H_0 = 0.045 m, d_3 = 0.0045 m, ε = 0.476, L = 0.18 m (L is the length of the permeable barrier). The streamline pattern reveals the characteristics of the fluid flow from the disperser into the collector through the porous medium. They illustrate the distribution of the filtrational flow in the porous medium, and the presence of slipping of liquid on the interfaces of the media. The fact that the flow distribution is nonuniform along the permeable barrier can be seen from the profiles of the transverse component of the velocity $\tilde{u}_2 = \varepsilon u_2$; thus, in the present process the ratio of the maximum value of $\tilde{u}_2$ to the minimum value is n = 1.91 at the entrance to the permeable barrier and 1.98 at the exit from it. This change is explained by redistribution of the pressure field in the disperser and collector. Pressure is restored in the dispenser, while in the collector the pressure drops in the direction of motion [14].

Some structural features of the flow in the dispenser and collector can be seen from the distribution of the longitudinal velocity $\tilde{u}_1 = \varepsilon u_1$. For example, in the collector the u_1 profiles stretch out in the zone near the axis. The value of k in the packed bed reaches 30% of the kinetic energy of the average motion. In the dispenser a strong outflow of fluid increases the turbulent energy near the axis. Behind the permeable barrier the kinetic energy of turbulence becomes collapsed, and this process continues with the motion of the liquid in the outflow channel. This explains the stretching out of the $\tilde{u}_1$ profiles in the collector. Such a deformation of the u_1 curves has been noted many times in experiments [17, 18] and is attributed to the turbulization of the under conditions of strong injection and suctioning.

The calculations showed that the length of the permeable barrier and Re affect the distribution of the filtrational velocity along the packed bed. For example, under otherwise equal conditions an increase in these quantities results in an increase in the degree of nonuniformity of the profiles of the transverse velocity component.

Thus the computed data obtained for a wide range of values of the process parameters give a correct physical description of the turbulent motion in apparatus with a stationary packed bed. The approach developed here provides a theoretical foundation for constructing an engineering method for calculating the aerodynamics of diverse constructions of reactors with a permeable barrier.

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